# **CHFEN 3553 Chemical Reaction Engineering**

*April 28, 2003; 1:00 PM – 3:00 PM* 

#### Answer all questions

1. A first-order reaction  $A \rightarrow B$  is taking place in a recycle reactor. The initial concentration is 4 mol/liter, the reactor volume is 200 liters and the volumetric flow rate is 20 liters/s. For a recycle ratio of 5, a conversion of 60% is obtained. This configuration is to be replaced with a CSTR-PFR combination. A 50-liter CSTR is followed by a PFR of unknown volume. What volume of PFR is required is achieve the same conversion as in the recycle reactor?

17 points

**Recycle Reactor** 

$$\frac{k\tau}{R+1} = \ln\left[\frac{C_{A0} + RC_{Af}}{(R+1)C_{Af}}\right]$$

$$C_{A0} = 4, X = 0.6, C_{Af} = 1.6, R = 5, \tau = 10$$

$$\frac{k\tau}{R+1} = \ln\left[\frac{C_{A0} + RC_{Af}}{(R+1)C_{Af}}\right] = \ln\left[\frac{4+5\cdot1.6}{(5+1)\cdot1.6}\right] = 0.2231$$

$$k = 0.1339 \,\text{s}^{-1}$$
CSTR first,  $\tau_{cstr} = 50/20 = 2.5s$ 

$$X_1 = \frac{k\tau}{1+k\tau} = \frac{0.1339\cdot2.5}{1+0.1339\cdot2.5} = 0.2508$$

PFR following the CSTR

$$k\tau = \ln \frac{1 - X_1}{1 - X_2} = \ln \frac{1 - 0.2508}{1 - 0.6} = 0.6276$$
  
$$\tau_{PFR} = 4.6866$$
  
$$V_{PFR} = 4.6866 \cdot 20 = 93.73 \text{ lit}$$

2. Rate versus concentration data for a reaction is given below. Find the order of the reaction and the reaction rate.

Con. (mol/liter)	1.5	1.3	1.1	0.9	0.7	0.5	0.3
Rate (mol/lit-sec)	28	20	12.8	7.7	4.1	1.8	0.5

16 points



n=2.52 k=10.1 3. An elementary irreversible liquid-phase reaction  $A + B \rightarrow C$  is carried out in a CSTR. A and B are fed at molar rates of 1.25 mol/s and 1 mol/s respectively, at a temperature of 300 K. The reactor is jacketed and the jacket temperature can be assumed to be 310 K. An agitator contributes a work of 20.9 kW to the reactor. The volumetric flow rate is 5 lit/s. Additionally:

$$H_{A}^{0}(298 K) = -20 \text{ kcal/mol } H_{B}^{0}(298 K) = -25 \text{ kcal/mol } H_{C}^{0}(298 K) = -60 \text{ kcal/mol}$$
$$C_{pA} = C_{pB} = 40 \frac{\text{cal}}{\text{mol} \cdot \text{K}}, \ C_{pC} = 55 \frac{\text{cal}}{\text{mol} \cdot \text{K}}$$

$$k = 0.01 \frac{\text{lit}}{\text{mol} \cdot \text{s}}$$
 at 300 K,  $U \cdot A = 75 \frac{\text{cal}}{\text{s} \cdot \text{K}}$ ,  $E = 8 \text{ kcal/mol}$ 

Determine the volume of the reactor for 60% conversion of A.

16 points

## Use equation 8-50 from the text $\pm$

$$\begin{aligned} \frac{Q-W}{F_{A0}} - X\Delta H_{Rx} &= \sum \theta_{i} \tilde{C}_{pi} (T-T_{i0}) \\ \Delta H_{Rx} &= -60 + 20 + 25 = -15 \text{ kcal/mol} \\ \frac{(U \cdot A \cdot (T_{a} - T) - \dot{W})}{F_{A0}} - X \left( \Delta H_{Rx} (T_{R}) + \Delta \hat{C}_{p} (T - T_{R}) \right) = \sum \theta_{i} \tilde{C}_{pi} (T - T_{i0}) \end{aligned}$$

$$T = \frac{F_{A0}X\left(-\Delta H_{Rx}(T_R) + \Delta \hat{C}_p \cdot T_R\right) + F_{A0}(\theta_A \tilde{C}_{pA} + \theta_B \tilde{C}_{pB})T_0 + UAT_a - \dot{W}}{F_{A0}(\theta_A \tilde{C}_{pA} + \theta_B \tilde{C}_{pB}) + UA + F_{A0}X\Delta \hat{C}_p}$$
  
$$\Delta \hat{C}_p = 55 - 40 - 40 = -25\frac{cal}{mol - K}$$
  
$$T = \frac{1.25 \cdot 0.6(15,000 - 25 \cdot 298) + 1.25 \cdot (1 \cdot 40 + 0.8 \cdot 40) \cdot 300 + 75 \cdot 310 + 5000}{1.25 \cdot (1 \cdot 40 + 0.8 \cdot 40) + 75 + 1.25 \cdot 0.6 \cdot (-25)}$$
  
$$T = 416.5 \text{ K}$$

$$\ln \frac{k_2}{k_1} = \frac{-E}{R} \left( \frac{1}{T_1} - \frac{1}{T_2} \right) = \frac{-8000}{1.987} \left( \frac{1}{416.5} - \frac{1}{300} \right)$$
  

$$k_2 = 0.4268$$
  

$$V = \frac{F_{A0}X}{-r_A} = \frac{C_{A0}vX}{kC_{A0}^2(1-X)(\theta_B - X)} = \frac{vX}{kC_{A0}(1-X)(\theta_B - X)}$$
  

$$C_{A0} = 1.25/5 = 0.25 \frac{\text{mol}}{\text{lit}}, v = 5 \frac{\text{lit}}{\text{s}}$$
  

$$V = \frac{5 \cdot 0.6}{0.4268 \cdot 0.25 \cdot (1 - 0.6)(0.8 - 0.6)} = 351.43 \text{ liters}$$

4. Mechanism of a catalytic reaction  $A \rightarrow B$  is shown below.

$$A + S \xrightarrow[k_{-A}]{k_{-A}} A \cdot S$$
$$A \cdot S \xleftarrow[k_{-S}]{k_{-S}} B \cdot S$$
$$B \cdot S \xleftarrow[k_{-D}]{k_{-D}} B + S$$

Write down the rates of adsorption, surface reaction and desorption and derive an effective rate when, surface reaction is rate controlling.

16 points

$$r_{AD} = k_A (p_A C_v - \frac{C_{A \cdot S}}{K_A})$$

$$\frac{r_{AD}}{k_A} = 0$$

$$r_S = k_S (C_{A \cdot S} - \frac{C_{B \cdot S}}{K_S})$$

$$r_D = k_D (C_{B \cdot S} - p_B C_v / K_{DB})$$

$$\frac{r_D}{k_D} = 0$$

$$C_{A \cdot S} = K_A p_A C_v$$

$$C_{B \cdot S} = \frac{p_B C_v}{K_{DB}} = K_B p_B C_v$$

Substitute into  $r_s$ 

$$r_{S} = k_{s}(K_{A}p_{A}C_{v} - \frac{K_{B}p_{B}C_{v}}{K_{S}}) = k_{s}(K_{A}p_{A}C_{v} - \frac{K_{A}K_{B}p_{B}C_{v}}{K_{S}K_{A}})$$

$$= k_{s}(K_{A}p_{A}C_{v} - \frac{K_{A}p_{B}C_{v}}{K_{P}}) = k_{s}K_{A}C_{v}(p_{A} - \frac{p_{B}}{K_{P}})$$

$$C_{t} = C_{v} + C_{A\cdot S} + C_{B\cdot S} = C_{v} + K_{A}p_{A}C_{v} + K_{B}p_{B}C_{v}$$

$$C_{v} = \frac{C_{t}}{(1 + K_{A}p_{A} + K_{B}p_{B})}$$

$$r_{s} = k_{s}K_{A}(p_{A} - \frac{p_{B}}{K_{P}}) \cdot \frac{C_{t}}{(1 + K_{A}p_{A} + K_{B}p_{B})}$$

$$r_{s} = \frac{k_{s}K_{A}C_{t}(p_{A} - \frac{p_{B}}{K_{P}})}{(1 + K_{A}p_{A} + K_{B}p_{B})}$$

- 5. A first-order reaction  $A \rightarrow 3B$  is taking place in a PBR. The particles are 10 mm in diameter and the intrinsic rate constant (k') is 0.8 lit/kg-cat-s. A conversion of 75% is desired. Feed at 4 mol/s, containing 40% A and 60% inerts enters the reactor at 127<sup>o</sup>C and 5 atmospheres. The engineer designing the reactor neglects to consider that there might be internal diffusion to consider.
  - a. What weight of the catalyst does the engineer pack the reactor with?
  - b. If the diffusion coefficient is 0.08 cm<sup>2</sup>/s ad bulk density of the catalyst is 2.8 kg/liter, what conversion would actually result with the catalyst packed?
  - c. What weight of the catalyst did he need to use to meet the design specifications of 75% conversion? Assume that the reactor operates at constant pressure.

18 points

### $\delta = 2, y_{A0} = 0.4, \varepsilon = 0.8$

PBR Equation without the effectiveness factor

$$F_{A0} \frac{dX}{dW} = -r_{A} = kC_{A} = kC_{A0} \frac{(1-X)}{(1+\varepsilon X)}$$

$$\frac{dX}{dW} = \frac{kC_{A0}}{F_{A0}} \frac{(1-X)}{(1+\varepsilon X)} = \frac{k}{v_{0}} \frac{(1-X)}{(1+\varepsilon X)}$$

$$\int_{0}^{X} \frac{(1+\varepsilon X)}{(1-X)} dX = \frac{k}{v_{0}} W = (1+\varepsilon) \ln \frac{1}{1-X} - \varepsilon X = 1.8953$$

$$Pv_{0} = F_{T_{0}} RT$$

$$5 \cdot v_{0} = 4 \cdot 0.082 \cdot 400 \implies v_{0} = 26.24 \frac{\text{lit}}{\text{s}}$$

$$\frac{0.8}{26.24} W = 1.8953$$

$$W = 62.17 \, kg$$

PBR with internal diffusion

$$\phi = R \sqrt{\frac{k\rho}{D}} = 0.5 \sqrt{\frac{0.8 \cdot 2.8}{0.08}} = 2.6457$$
$$\eta = \frac{3}{\phi^2} \left(\frac{\phi}{\tanh \phi} - 1\right) = 0.5912$$
$$= \frac{3}{2.6457^2} \left(\frac{2.6457}{0.9878} - 1\right) = 0.7193$$
Conversion with 62.17 kg

$$\frac{\eta k}{v_0} W = (1+\varepsilon) \ln \frac{1}{1-X} - \varepsilon X$$
  
$$\frac{0.7193 \cdot 0.8}{26.24} 62.17 = (1+0.8) \ln \frac{1}{1-X} - \varepsilon X = 1.3634$$
  
$$X = 0.66$$

Weight to acheive 75% conversion = 62.17/0.7193=86.4 kg

6. The residence time distribution function for a reactor is given below. The reaction is ½ order,  $C_{A0}=1$  mol/lit and the rate constant is  $2 \frac{\text{mol}^{1/2}}{\text{lit}^{1/2} - \text{min}}$ . Determine the conversion in the reactor using the segregated-flow model.



17 points

## The performance equation is

$$\frac{C}{C_0} = \int_0^\infty \left(\frac{C}{C_0}\right)_{\text{batch}} E(t) dt$$

For a general n-th order reaction

$$(n-1)kC_0^n t = \left(\frac{C}{C_0}\right)^{1-n} - 1$$

For 1/2 order reaction,

$$\left(-\frac{1}{2}\right)2(1)t = \left(\frac{C}{C_0}\right)^{\frac{1}{2}} - 1$$
$$\left(\frac{C}{C_0}\right)_{\text{batch}} = (1-t)^2$$
$$\frac{C}{C_0} = \int_0^2 (1-t)^2 (0.5)dt = \frac{0.5}{3}(1-t)^{\frac{30}{2}} = \frac{1}{6}(1+1) = \frac{1}{3}$$
$$\overline{X} = 0.66667$$