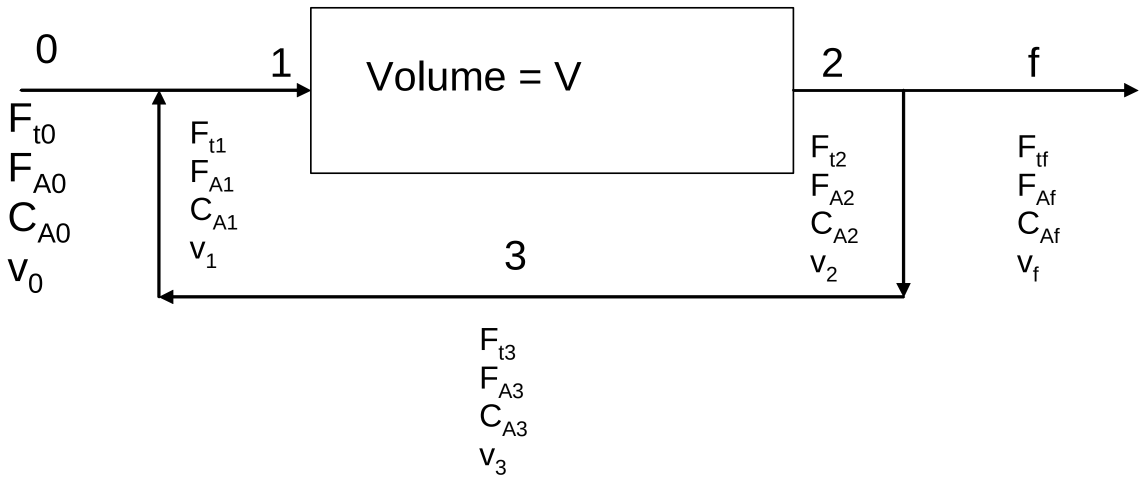
Stream numbers are indicated at the top



For incremental conversion in a PFR, the mole balance reduces to:

$$\frac{V}{F_{A0}{}'} = \int_{X_{A1}}^{X_{A2}} \frac{dX_A}{(-r_A)}$$

Where, $F_{A0}{}'$ refers to unconverted feed

Consider the reaction

$$aA + bB \rightarrow cC + dD$$

$$\partial = \left(\frac{c}{a} + \frac{d}{a} - 1 - \frac{b}{a}\right)$$

$$R = \frac{v_3}{v_f} = \frac{v_3 \cdot C_{A3}}{v_f \cdot C_{Af}} = \frac{F_{A3}}{F_{Af}}$$

Product Stream:
$$F_{Af} = F_{A0}(1 - X_2)$$
$$F_{tf} = F_{t0} + \partial(F_{A0} - F_{Af})$$

$$F_{A1} = F_{A0} + RF_{Af} = F_{A0}[1 + R(1 - X_2)]$$
$$F_{t1} = F_{t0} + RF_{tf}$$
$$v_1 = v_0 + Rv_f$$

Inlet Stream: $C_{A1} = \frac{F_{A1}}{v_1} = \frac{F_{A0} + RF_{Af}}{v_0 + Rv_f}$

$If\ v_0 = v_{f,}$

$$C_{A1} = \frac{C_{A0} + RC_{A2}}{1 + R}$$

$$C_{A2} = C_{Af}$$

Reactor Outlet Stream: $F_{A2} = (R+1)F_{Af} = F_{A0}(R+1)(1 - X_2)$

Recycle Stream: $F_{A3} = RF_{Af}$

In general, inside the reactor: $F_t = F_{t1} + \partial(F_{A1} - F_A)$ based on the definition of ∂

In general, for any system with volume change

$$V = V_0(1+\varepsilon_A X_A)$$

$$N_A = N_{A0}(1-X_A)$$

$$C_A = \frac{N_A}{V} = \frac{N_{A0}(1-X_A)}{V_0(1+\varepsilon_A X_A)} = \frac{C_{A0}(1-X_A)}{(1+\varepsilon_A X_A)}$$

$$X_A = \frac{1-\frac{C_A}{C_{A0}}}{1+\varepsilon_A(\frac{C_A}{C_{A0}})} \Rightarrow X_{A1} = \frac{1-\frac{C_{A1}}{C_{A0}}}{1+\varepsilon_A(\frac{C_{A1}}{C_{A0}})}$$

$$C_{A1} = \frac{F_{A1}}{v_1} = \frac{F_{A0}+F_{A3}}{v_0+Rv_f} = \frac{F_{A0}+RF_{A0}(1-X_{Af})}{v_0+Rv_0(1+\varepsilon_A X_{Af})} = C_{A0}(\frac{1+R(1-X_{Af})}{1+R(1+\varepsilon_A X_{Af})})$$

$\dfrac{C_{A1}}{C_{A0}} = \left(\dfrac{1+R(1-X_{Af})}{1+R(1+\varepsilon_A X_{Af})}\right)$ Use this to find X_{A1}

$$X_{A1} = \left(\frac{R}{R+1}\right)X_{Af}$$

General design equation for the recycle reactor:

In terms of unconverted feed $F_{A0}' = (1+R)F_{A0}$

$$\frac{V}{F_{A0}} = (1+R)\int_{\left(\frac{R}{R+1}\right)X_{Af}}^{X_{Af}} \frac{dX_A}{(-r_A)}$$

Please also look at homework solutions for relationships at constant volume.