Assignment 8 – Analytical Solutions

1. Problem 6-7

$$A \xrightarrow{k1} B \xrightarrow{k2} C$$

$$\frac{dC_A}{dt} = -k_1 C_A \qquad A: alcohol\ in\ GI\ tract$$

$$Initial\ Condition: C_A = C_{Ao}\ at\ t = 0$$

$$Thus,$$

$$C_A = C_{A0} e^{-k_1 t}$$

$$\frac{dC_B}{dt} = -k_2 + k_1 C_A \qquad B: alcohol\ in\ blood$$

$$Initial\ Condition: C_B = 0\ at\ t = 0$$

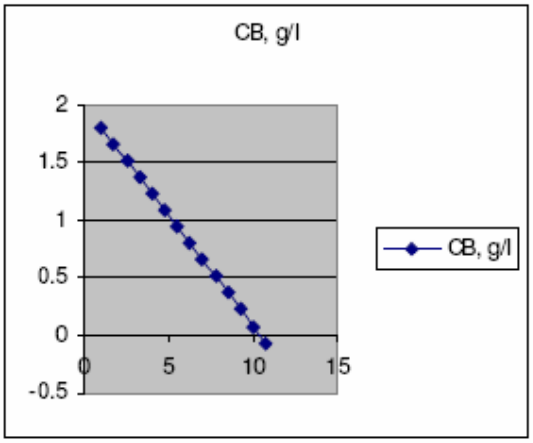
$$Thus,$$

$$C_B = C_{A0}(1 - e^{-k_1 t}) + C_{B0} - k_2 t$$

a. Two tall martinis = 80 g EtOH
Body fluid = 40 L
C_{A0} = 80/40 = 2 g/l

Thus, $C_B = 2(1 - e^{-10_1 t}) + 0 - 0.192t$, t in hrs

t	CB, g/l
1	1.807909
1.75	1.664
2.5	1.52
3.25	1.376
4	1.232
4.75	1.088
5.5	0.944
6.25	0.8
7	0.656
7.75	0.512
8.5	0.368
9.25	0.224
10	0.08
10.75	-0.064



For U.S., CB = 1 g/l => t = 5.21 h

b. For Sweden, CB = 0.5, thus t = 7.8 h
c. For Russia, t = 10.4 h
d. Drinks taken ½ hr apart.

For the 1st ½ hr:

$C_A = 1e^{-10t}$

$C_B = 1(1 - e^{-10_1 t}) + 0 - 0.192t$

At the end of 1st ½ hr: t = 0.5

$C_{A,1/2} = e - 5 = 6.74e - 3 \ \ g/l$

$C_{B,1/2} = (1 - e^{-5}) - 0.192 * 0.5 = 0.8973 \ \ g/l$

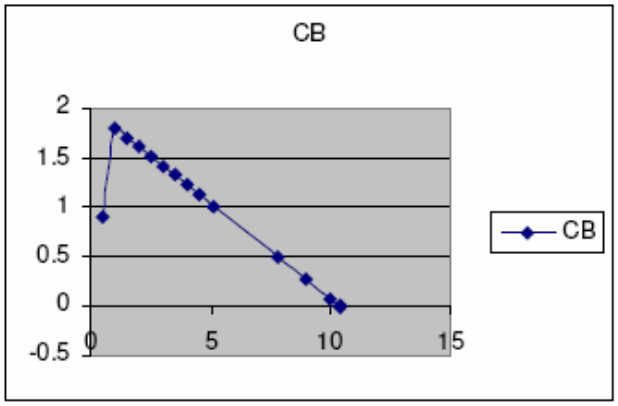
After the 1st ½ hr:

$C_A = C_{A,1/2} e^{-k_1(t-0.5)}$

$C_B = C_{A,1/2}(1 - e^{-k_1(t-0.5)}) + C_{B,1/2} - k_2(t - 0.5)$

t	CB
0.5	0.8973
1	1.794562
1.5	1.705255
2	1.6093
2.5	1.5133
3	1.4173
3.5	1.3213
4	1.2253
4.5	1.1293
5.15	1.0045
7.8	0.4957
9	0.2653

10	0.0733
10.35	0.0061
10.4	-0.0035



For CB = 1g/l (U.S.), t = 5.15 h
= 0.5 g/l (Swe), t = 7.8 h
= 0 g/l (Rus), t = 10.4 h

e. Uniform rate of consumption = 2 g/l/h

For the 1st hour:

Rewrite basic equations:

$$\frac{dC_A}{dt} = 2 - k_1 C_A$$

$$Initial\ Condition: C_A = 0\ at\ t = 0$$

Thus,

$$C_A = \frac{2}{k_1}(1 - e^{-k_1 t})$$

$$\frac{dC_B}{dt} = -k_2 + k_1 C_A$$

$$Initial\ Condition: C_B = 0\ at\ t = 0$$

Substituting for C_A,

$$C_B = \frac{2}{k_1}(e^{-k_1 t} - 1) - (k_2 + 2)t$$

At the end of the 1st hour:

$$C_{A,1} = \frac{2}{10}(1 - e^{-10}) = 0.2 \ g/l$$

$$C_{B,1} = \frac{2}{10}(e^{-10} - 1) - (0.192 + 2) = 1.608 \ g/l$$

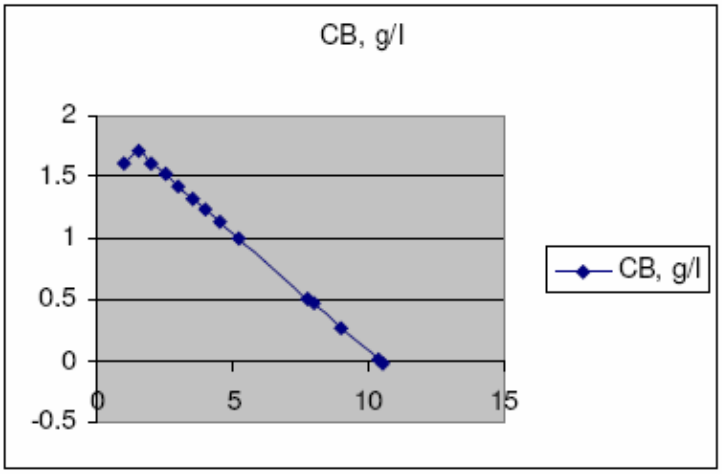
After the 1st hour:

$$C_A = C_{A,1} e^{-k_1(t-1)}$$

$$C_B = C_{A,1}(1 - e^{-k_1(t-1)}) + C_{B,1} - k_2(t-1)$$

Thus,

t	CB, g/l
1	1.608
1.5	1.710652
2	1.615991
2.5	1.52
3	1.424
3.5	1.328
4	1.232
4.5	1.136
5.21	**0.99968**
7.8	0.5024
8	0.464
9	0.272
10.4	0.0032
10.5	-0.016



Thus, CB =1(US), t = 5.21 h

f. 60 g immediately => CA0 = 1.5 g/l

$$C_B = C_{A0}(1 - e^{-k_1 t}) + C_{B0} - k_2 t = 1.5(1 - e^{-10t}) - 0.192t$$

CB = 1 g/l at **t=0.115 h** and at t=2.6h