

An Empirical Model for Predicting Diffusion Coefficients in Silicate Minerals

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An empirical model describing the diffusion kinetics of oxygen in silicate minerals under hydrothermal conditions has been established for temperatures between 773 and 1073 Kelvin at 100 megapascals of water pressure. The equation, $\log D = \alpha + (\beta/T) + [\gamma + (\delta/T)]Z$, where D is the diffusion coefficient, α , β , γ , and δ are constants, T is the Kelvin temperature, and Z is the total ionic porosity, may be used to predict diffusion coefficients, in most cases to within the reported experimental reproducibility of a factor of 2. For oxygen diffusion, $\alpha = -2$, $\beta = -3.4 \times 10^4$ K, $\gamma = -0.13$, and $\delta = 6.4 \times 10^2$ K, for D in square centimeters per second. Limited data for the diffusion of argon in silicates suggest that the model describes this system as well.

THE POTENTIAL FOR SOLVING A WIDE range of problems in the earth sciences has motivated much research on rates of diffusion in minerals and glasses. Applications include the interpretation of geothermometers and geobarometers, disturbed geochronologic systems, and thermal histories of rocks. These applications, and others, involve a number of diffusing species in a variety of different minerals. A method of predicting diffusion coefficients without having to measure them (which is difficult) would clearly be of value.

A method of estimating diffusion coefficients (D) to approximately an order of magnitude by the compensation law, which relies on an observed correlation between $\ln D_0$, the natural logarithm of the preexponential factor in the Arrhenius diffusion relation $D = D_0 \exp(-Q/RT)$, and Q , the activation energy, has been reported (1) (R is the gas constant and T is the absolute temperature). The approach makes use of a compensation relation for both the species and the mineral of interest, which requires considerable experimental data. In addition, diffusion compensation rests on the assumption that the diffusion mechanism is the same for all the minerals and species being considered (2).

An alternative approach was suggested by Dowsy (3): that some measure of the closeness of packing of the ions in a mineral is likely to have a strong influence on diffusion through a given structure. He showed that there was a qualitative relation between anion porosity (fraction of space in the unit cell not occupied by anions) and diffusivity,

Table 1. Calculated total ionic porosities. The upper group of minerals consists of those for which experimental data are available for O diffusion under hydrothermal conditions (see text for references). The lower group consists of those for which diffusivity predictions are made in this report. All volumes are expressed in cubic angstroms; V_1 is the sum of the volumes of cations and anions in the unit cell (5); V_C is the volume of the unit cell at 24°C (6), and Z is the total ionic porosity at 24°C. V_0 and dV/dT are derived from thermal expansion data and may be used to scale Z to the temperature of interest (7). dV/dT for β -quartz equals 0 because no significant change occurs in the cell dimensions between the α - β transition temperature and $\sim 1000^\circ\text{C}$.

Mineral	V_1 (\AA^3)	V_C (\AA^3)	Z (%)	V_0 (\AA^3)	dV/dT ($\text{\AA}^3 \text{K}^{-1}$)
Albite	371.93	665.32	44.1	657.29	0.0203
Anoorthite	743.20	1339.79	44.5	1333.24	0.0178
Biotite	297.58	492.38	39.6	490.23	0.0136
Dioptside	287.80	437.88	34.3	433.01	0.0134
Hornblende	561.59	911.97	38.4	901.59	0.0285
Muscovite	579.30	937.77	38.2	931.02	0.0295
Orthoclase	398.18	719.93	44.7	713.45	0.0165
Phlogopite	296.54	496.77	40.3	494.60	0.0137
α -quartz	66.27	112.98	41.3	102.25	0.0180
β -quartz	66.27	118.12	44.3	118.12	0.0000
Richterite	562.98	900.54	37.5	899.76	0.0304
Tremolite	561.97	907.10	38.0	895.16	0.0320
Almandine	1114.45	1532.81	27.3	1516.40	0.0424
Andalusite	224.17	343.21	34.7	362.92	0.0156
Andradite	1177.51	1749.69	32.7	1733.57	0.0467
Forsterite	188.94	290.34	34.9	284.86	0.0132
Grossularite	1169.08	1664.01	29.7	1648.14	0.0431
Nepheline	403.53	729.78	44.7	712.66	0.0413
Pyrope	1106.07	1505.06	26.5	1490.51	0.0417
Spessartite	1123.98	1568.98	28.4	1551.93	0.0472
Sphencite	241.02	369.62	34.8	365.96	0.0087
Zircon	182.77	260.49	29.8	258.80	0.0041

O diffusivity predicted with model

Experimental O diffusion data available

as measured by the relative ease of reequilibration of O isotopes in natural systems (3). The recent acquisition of additional experimentally determined diffusion data has made it possible to develop this approach further.

The basis for our model is an observed linear relation between $\log D$ at a particular temperature, for O diffusion in silicates, and Z . These minerals represent a range of structures from framework to single-chain silicates. The rate for the fastest transport directions from framework to single-chain silicates. The rate for the fastest transport directions from framework to single-chain silicates. The rate for the fastest transport directions from framework to single-chain silicates.

$$Z = [1 - (V_1/V_C)] \times 100 \quad (1)$$

observed for these micas (13). The data in Fig. 1 indicate that larger and roughly similar ionic porosities for the micas would be required to obtain agreement between the model and the experimental data. No calculated ionic porosity has been found to satisfy the data, however, particularly the variation of D as a function of temperature.

Effective ionic porosities for the micas can be determined by projecting the experimental values onto the regression line for $\log D_T$ versus Z that is obtained from the eight other minerals (20). Are the ad hoc ionic porosities determined in this manner reasonable? One way to test this hypothesis is to assume that a structural parameter, like the ionic porosity, would behave the same for argon diffusion as for O diffusion.

We applied the model to available argon diffusion data (21), obtained under similar experimental conditions, for orthoclase, phlogopite, hornblende, and biotite, using the effective ionic porosities for the micas

Fig. 2. Arrhenius plot of model versus experimental values of the diffusion coefficient for O diffusion. Solid lines are experimental data (see text for references). Dashed lines are model values from Eq. 5. Abbreviations as in Fig. 1.

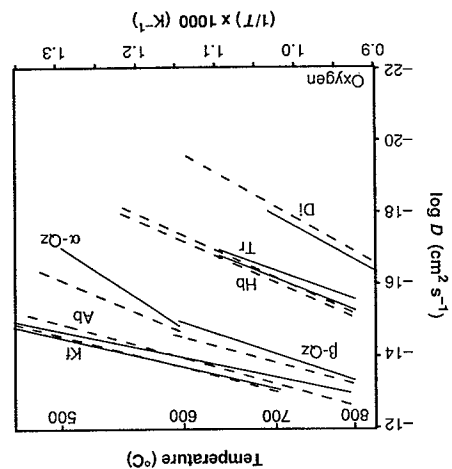
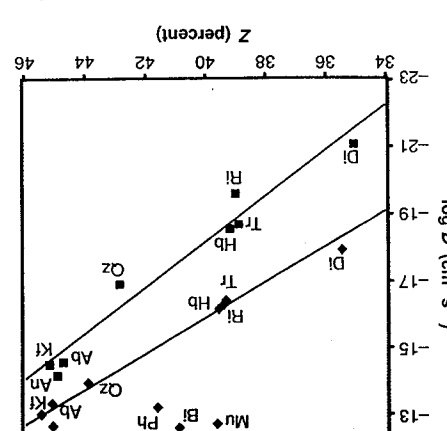


Fig. 1. Plots of $\log D$ versus Z for O at 500° and 700°C (Fig. 1) are representative of O diffusion behavior throughout the experimental temperature range. Apparently as a result of their large interlayer spacings, the micas all have anomalously. A possible approach to modeling the micas is discussed below.



the micas have proven difficult to model (3). The most likely reason for this is that large interlayer spacings, and phlogopite have muscovite, biotite, and orthoclase. In addition, there is a much larger than for the interlayer spaces are much larger than mica bulk ionic porosities. For mal expansion occurs preferentially along the c -axis, and most of the change occurs in the interlayer space (7). Exchange controlled by these large spacings can account for the rapid rates of O exchange that have been

observed for these micas (13). The data in Fig. 1 indicate that larger and roughly similar ionic porosities for the micas would be required to obtain agreement between the model and the experimental data. No calculated ionic porosity has been found to satisfy the data, however, particularly the variation of D as a function of temperature. Effective ionic porosities for the micas can be determined by projecting the experimental values onto the regression line for $\log D_T$ versus Z that is obtained from the eight other minerals (20). Are the ad hoc ionic porosities determined in this manner reasonable? One way to test this hypothesis is to assume that a structural parameter, like the ionic porosity, would behave the same for argon diffusion as for O diffusion.

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where D_T is the diffusion coefficient at temperature T , A is the γ -intercept, B the slope of the regression line (15), and Z the total ionic porosity. The relations for O diffusion in silicates also change systematically as a function of temperature. Because the data for individual minerals obey an Arrhenius relation, the intercepts A and the slopes B can be expressed as linear functions of the inverse Kelvin temperature:

$$\log D = \alpha + (\beta/T) \quad (3)$$

$$B = \gamma + (\delta/T) \quad (4)$$

Substitution of these expressions for A and B in Eq. 2 yields

$$\log D = \alpha + (\beta/T) + \{\gamma + (\delta/T)\}Z \quad (5)$$

This single equation, where D is a function of temperature and total ionic porosity only, provides a model for O diffusion in silicates minerals under the experimental conditions for which the data are known. On the basis of a minimal amount of experimental data to determine the constants for the model, approximate values of the diffusion coefficient for O in other minerals can be generated.

For O diffusion, based on the data for eight minerals, the values of the constants are $\alpha = -2 \pm 1$, $\beta = -(3.4 \pm 0.3) \times 10^4 \text{ K}$, $\gamma = -0.13 \pm 0.03$, and $\delta = (6.4 \pm 0.6) \times 10^2 \text{ K}$ (16). The Arrhenius plots comparing the experimental data for several minerals with the values predicted by the model (Fig. 2) are within, or closely approach, the estimated experimental reproducibility of a factor of 2 reported for the diffusion coefficients with the exception of α -quartz. The experimental data for α -quartz are the least certain of all the data shown and indicate an activation energy of $68 \pm 22 \text{ kcal mol}^{-1}$ (10); the model predicts 45 kcal mol^{-1} . For the other data (without α -quartz), 1 SD of the absolute values of the difference between the measured and the predicted activation energies is 3 kcal mol^{-1} , comparable to

Fig. 3. Model versus experimental Arthenius plots for argon diffusion. Solid lines represent experimental values (see text for references). Dashed lines are model values. Model values for biotite (Bi) and phlogopite (Ph) were calculated from Eq. 5 and by use of effective ionic porosities determined from the O diffusion data (see text). Or, orthoclase.

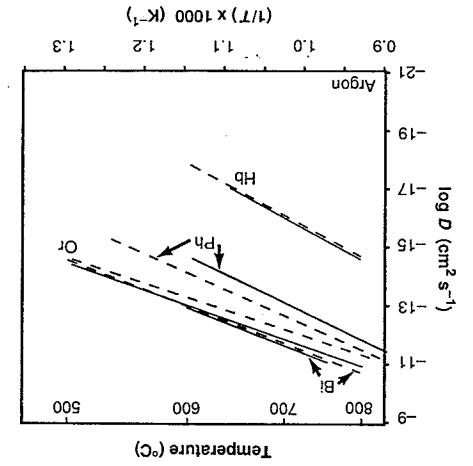
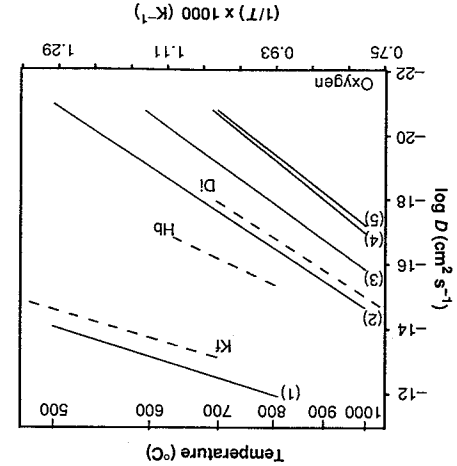


Fig. 4. Predicted Arthenius relations for various minerals. Values of the total ionic porosity were corrected for the effects of thermal expansion. Minerals: (1) nepheline; (2) forsterite; (3) andradite; (4) grossularite; (5) zircon. Data for potassium feldspar (Kf), hornblende (Hb), and diopside (Di) are shown as dashed lines for comparison with Fig. 2.



With Eq. 5, the constants given above, Eq. 1, and the data in Table 1, Arthenius relations for O diffusion under hydrothermal conditions can be predicted for minerals for which experimental data (at the conditions of the reference data) are lacking (Fig. 4). Many minerals of potential interest may have slow diffusivities for O (Fig. 4 and Table 1), which makes experiments difficult and a predictive model particularly valuable. The fit of the model to the data indicates that, on the level of individual minerals, Eq. 5 is a reasonable approximation of the Arthenius relation $D = D_0 \exp(-Q/RT)$. Therefore D_0 and Q must both be affected by the total ionic porosity. Intuitively, activation energy is clearly linked to the ionic porosity when viewed as an energy barrier that must be overcome for diffusion to occur. Lower porosity (that is, closer packing) implies a higher energy barrier. The dependence of the preexponential factor is more complicated, and its relation to ionic porosity is less clear. The jump distance, λ , one factor given in expressions for D_0 (23), is likely to be related to the closeness of packing, but there is no apparent, simple relation between Q and Z (7).

Treating silicate minerals as a collection of more or less closely packed ionic spheres is clearly a simplification. The fit of the model to the 100-MPa hydrothermal experimental data for O, however, is remarkably good. Our observations concerning argon diffusion and single-crystal data for dry O diffusion suggest that Eq. 5, with different coefficients, may apply in those cases as well. The total ionic porosity is a property of the material structure, which does not change with the diffusing species, and therefore should influence cation diffusion as well. There are, however, insufficient data

on cation diffusivities to permit a test of the model. We suggest that this model provides a framework in which relative diffusivities can be rationalized and may also serve as a basis for more fundamental diffusion models. If this approach can be extended to the diffusion of additional species (cations?) and to other experimental conditions it could prove to be a powerful tool to guide and extend experimental investigations.

A substantial body of data for O diffusion under dry conditions at 1-atm total pressure is available. A linear relation between activation energy and anion porosity has been reported for a limited portion of these data (24). Trends in $\log D_T$ versus Z are evident in the dry data. There appears to be a fundamental dichotomy, however, between data from single-crystal experiments. The only mineral for which direct comparison is possible is anorthite. The bulk exchange data (25) imply faster diffusion and a higher activation energy than the single-crystal data (26). Because volume diffusion is likely to be the slowest process measured in these experiments, the lower values are more likely to be correct. The single-crystal data for anorthite (26), β -quartz (27), and forsterite (24, 25, 29) all lie above this trend. Further experimental work is needed to determine which measured values to use in testing the model.

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5. Ionic radii used to calculate the volume of the ions are effective ionic radii from R. D. Shannon and C. T. Prewitt, *Acta Cryst.* 25, 925 (1969). The following conventions were used to calculate V_i : (i) O radius was set equal to 1.38 Å; (ii) all transition elements were assumed to be in the high-spin state; (iii) no changes in coordination or ionic radii were allowed to occur with increasing temperature. The constants for the model can be determined with any rational set of ionic radii. Different sets of radii yield different values of V_i , ionic porosity, and constants. The linear relation holds, however, and the resulting predicted diffusivities differ only on the order of a factor of 10. In order to use our values of the constants to predict diffusivities, the ionic radii and conventions we have specified should be employed.

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10. B. J. Gillett and R. A. Yund, *J. Geophys. Res.* 89, 4039 (1984).

11. J. R. Farver and B. J. Gillett, *Geochim. Cosmochim. Acta* 49, 1403 (1985).

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15. The physical significance of the constants A and B is apparent from consideration of Fig. 1. The intercept, A , is simply $\log D_T$ at zero porosity and is an extremely small number, in accord with physical intuition. The slope, B, is the dependence of $\log D_T$ on the total ionic porosity, and it decreases with an increase in temperature. The temperature-dependent relation for B (Eq. 4) indicates that B approaches γ as T approaches ∞ . Because γ is close to zero (-0.13), this relation is again in accord with physical intuition, that is, the rate of exchange is independent of the structure at infinite temperature. The magnitude of δ in Eq. 3 (6.4×10^2) dictates that there will always be a finite dependence on porosity for O diffusion in silicates under subsolidus, hydrothermal conditions.

16. Calculation of the constants involves two least-squares fits, the first for the experimental data ($\log D_T$) versus Z , and the second for the constants A and B versus the inverse Kelvin temperature (be-

